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WAVE GENERATION NEAR THE OUTER BOUNDARY OF THE MAGNETOSPHERE*

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ABSTRACT

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It is shown that certain plasma instabilities can explain the enhanced and fluctuating magnetic field observed in the layer of disturbed plasma lying between the magnetosphere and the solar wind. Unstable ion waves grow into non-linear behavior during approximately the foremost one-fourth of that region. The resulting turbulence, with perhaps some residual oscillation at the frequency of the unstable waves, persists through the rest of the region.

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I. INTRODUCTION

This paper describes plasma instabilities that are expected to contribute to the enhanced and fluctuating magnetic field observed in the layer of disturbed plasma lying between the magnetosphere and the solar wind. The data are not as yet adequate to establish whether the mechanisms described here are the sole or principal cause of the fluctuations, but the instabilities in question are certain to play some role in the behavior of the solar plasma impinging on the magnetosphere.

Axford (1962) and Kellogg (1962) presented similar reasonable pictures, based on experimental evidence, of the flow of the solar wind in the neighborhood of the earth. According to Kellogg (1962) there is a thin shock at the edge of the solar wind region, behind which there is a turbulent or disordered region about 4 earth radii thick on the sunward side. Solar plasma and magnetic field enter this region after being compressed and slowed in the shock, and are swept around toward the night side, expanding and accelerating in the process. It remains to explain the spectrum of irregularities observed in the disordered region (Sonett, Smith and Sims 1960; Cahill and Amazeen 1962; Heppner, Ness, Skillman, and Scearce 1961).

Dessler (1962) showed on the basis of considerable observational evidence that the inner boundary, between the disordered region and the magnetosphere, is stable. Therefore the fluctuating magnetic fields cannot result from the outward propagation of disturbances generated at this surface. It is possible that waves are generated in the shock front itself. Collisionless shock theories (Davis, Lust, and Schlüter 1958; Morawetz 1961; Auer, Hurwitz and Kilb 1962) are not conclusive as to what waves if any should be present in the shocked region, nor is the distinction between the shock front and the shocked region perfectly clear in these theories. Waves are generally produced in what I shall call a shock front, whose thickness is a few ion Larmor radii. These waves then decay over a much greater distance by means of collisions or non-linear interactions. The present paper is a contribution to collisionless shock theory in the sense that it gives a mechanism for the generation of waves behind the shock, the waves eventually leading through non-linear behavior to entropy production. Since this paper is not primarily concerned with collisionless shock theory, no attempt has been made to explain the details of what happens in the thin shock front. It is then still a possibility that some processes in the shock front itself generate some of the observed irregularities.

The explanation advanced here rests on the pressure anisotropy that should be generated in the shock front (Kellogg 1962; Auer et al 1962). The solar magnetic field makes an angle of more than 45° with the solar wind flow direction (McKracken 1962). Since the particle collision rates are quite small, the compression that occurs when the solar wind is slowed in the shock front should result in an excess of pressure perpendicular to the magnetic field lines ($p_{\perp}$) as compared to the pressure along the field lines ($p_{\parallel}$). (Some two dimensional randomization mechanism is presumed to operate in the shock front so that the gas is describable in terms of a pressure and temperature perpendicular to the field lines and a different temperature and pressure parallel to them. The precise distribution of particle velocities used is given in the next section.) The anisotropy of the plasma behind the shock front leads to instability with respect to transverse waves (Harris 1961; Noerdlinger 1963).*

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* Harris' results are inapplicable because he assumed zero pressure along the magnetic field, which greatly changes the stability properties. He also did not find the growth rates of most of the waves. Noerdlinger's published results are not quite adequate either, because they are based on a fictitious ion mass equal to either the electron mass or infinity. Accordingly, the results presented here are obtained from more recently performed machine calculations.

According to the set of values adopted for the solar wind velocity and the average magnetic field strength in the solar wind, there is some variation in the properties of the unstable waves. This variation leads to different ways of relating wave properties to observables. For example, if the waves grew somewhat more slowly than in the examples given here, they themselves would be observable over most of the disturbed region. It so happens that in the examples worked out the growth passes well into the turbulent non-linear regime during the first 5000 km or so of travel through the disordered region. Therefore field fluctuations in most of the region would be due to the passage of turbulent eddies past the probe rather than to the direct presence of the waves.

Two sets of plasma conditions are used, corresponding to the use of data from Pioneer V, Explorer X and Explorer XII with (Case "A") or without (Case "B") the preliminary results from Mariner II. The Mariner II preliminary data indicate higher plasma velocity and lower magnetic field in the solar wind than did previous measurements. This implies that previous vehicles did not get completely clear of the earth's influence as an obstruction in the solar wind. The properties of the growing waves found do not differ by large factors in the two cases. Approximate scaling laws are given to adapt the calculations to further changes in initial conditions but the real situation is so much more complicated than the one treated here that the results cannot be expected to match observations very closely. For example, no account is taken here of the curvature of the shock front nor of the gradual expansion of the plasma as it moves toward the night side, nor is a model of the shock worked out.

Section II presents estimates of the plasma pressure, density, and anisotropy, and of the magnetic field values. In Section III the unstable waves expected in such a plasma are discussed and compared with experimentally observed irregularities.

II. PLASMA CONDITIONS

An attempt will now be made to estimate the density, temperature, and degree of anisotropy of the solar plasma just after rapid compression, as well as the strength of the magnetic field imbedded in it. New experimental data are appearing frequently and recent data are often subject to revision under more detailed reduction; hence this paper makes no pretence of giving definitive values. There is also some fluctuation in the values, especially during magnetic storms. Such fluctuations are ignored.

In all cases the number density of electrons and of protons in the solar wind at the orbit of earth will be taken as 10 cm^{-3}. This figure is consistent with experimental observation (Bridge, Dilworth, Lazarus, Lyon, Rossi and Scherb 1961) and with theory (Parker 1960). One estimate of the solar wind velocity and ambient magnetic field (designated Case A) will be based primarily on the preliminary Mariner II results (Snyder 1962; Davis 1962). For comparison, another set of values will also be considered (Case B) that are based solely on older data (Bridge et al 1961; Cahill and Amazeen 1962; Heppner et al 1961; Sonett et al 1960).

Case A

For Case A the solar wind velocity is about 500 km/sec. and its temperature is of order 2×10^5 deg. (Snyder 1962). The extent to which the plasma is compressed and rendered anisotropic as it passes through the thin shock front is difficult to determine accurately. The usual fluid shock theory for a gas with $c_p/c_v = \gamma$ would give a maximum compression ratio $Q = \rho_2/\rho_1$ of 3 if $\gamma = 2$ (two-dimensional gas). Even if $\gamma = 5/3$ the upper limit for the compression ratio is 4. On the other hand, magnetic field data from Mariner II suggest a much larger compression ratio. Davis

(1962) measured the magnetic field in the solar wind to be roughly 7γ*. If the lines of force make an angle θ with the shock front on the upstream side, the field is increased by a factor $\sin\theta\ (1 + Q^2\cot^2\theta)^{1/2}$ when the plasma is compressed. Theory (Parker 1958) and some experimental evidence (McKracken 1962) show that the field lines are wrapped into rather flat spirals around the sun. Therefore θ will be taken to be 40° or less. For $Q > 2$ and $\theta < 40^\circ$ the factor by which the field is increased is approximated to within 11% by the function $Q\cos\theta$, the approximation improving rapidly as Q or $\cos\theta$ increases. Cahill and Amazeen (1962) measured the field in the disordered region on the sunward side of the magnetosphere. They obtained an average value of 30 to 40 γ. Heppner et al (1961) made similar measurements out to 38.5 earth radii in a direction about 45° from the anti-solar direction. During periods (such as at 22-27 earth radii) when their Explorer X probe was clearly inside the disordered region the magnetic field was about 30 γ. The generally lower field values observed by Heppner et al are reasonable in view of the expansion the plasma undergoes as it moves around toward the night side (Kellogg 1962).

Neither Heppner's nor Cahill's values for the magnetic field in the disordered region are compatible with Davis' value of 7γ before compression, since $Q\cos\theta$ can hardly exceed 3. It then seems fair to assume that Q attains nearly its maximum value of 3 and that some other mechanism is responsible for increasing the magnetic field <u>subsequent</u> to compression. The unstable hydromagnetic waves discussed in this paper are in fact a likely source for much of the additional field. The field is observed to fluctuate with a spectrum peaked at about 0.1 c.p.s. (Sonett 1960); the integral of Sonett's power spectrum is about 10^{-7} $(\text{gauss})^2$, corresponding to a r.m.s. field component perpendicular to the satellite spin axis of about 33 γ. This accounts for

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* here γ is a unit of magnetic field equal to 10^{-5} gauss.

nearly all the field observed by Sonett or later by Cahill (1962) and by Heppner (1961). In other words, the oscillating part of the field is the major part of it in the disordered region. This is consistent with the presence of either large amplitude hydromagnetic waves or turbulent eddies containing trapped magnetic field and sweeping past the vehicle. Both types of behavior result from the instabilities described in this paper. In Case A, which is now under consideration, the stated assumption that Q is nearly three leads to a magnetic field of 15 γ immediately after compression. ($Q \approx 3$, $\theta \approx 40^\circ$, $\cos\theta \approx 0.77$). If θ were zero essentially all the streaming energy would be converted into random energy perpendicular to the magnetic field lines. With nonzero θ , some of this energy goes instead into random or thermal energy along the field lines. It is hard to tell just how the energy divides, because the lines of force make a somewhat flatter angle with the shock front downstream than upstream and the compression is irreversible. (In a reversible adiabatic anisotropic compression the adiabatic gas law could be applied separately to the longitudinal and transverse parts of the pressure.) So long as θ is less than 45° it is evident that $p_\perp$ exceeds $p_\parallel$; for the present purposes the ratio $\Lambda = p_\perp / p_\parallel$ will be taken as about 2.

In summary, it will be assumed in Case A that the plasma has been anisotropically compressed so that its number density is $n_o = 30$ particles/cc. of each species, its thermal or random energy perpendicular to the field lines corresponds to 5×10^6 deg., and the magnetic field is 15 γ . The magnetic pressure $B^2/8\pi$ is then about 9×10^{-10} dyne/cm^2 while $p_\perp = 4 \times 10^{-8}$ dyne/cm^2 and $p_\parallel \approx \frac{1}{2} p_\perp$. The ions and electrons contribute equally to the pressure.

Other quantities of interest are the Debye length $\lambda_D \approx 4 \times 10^3$ cm., the plasma angular frequency $\omega_p \approx 3 \times 10^5$ radians/sec., and the electron collision frequency $\nu_c \approx 10^{-7}$ sec^{-1} (Spitzer 1956). The ion gyro angular frequency Ω is 1.5 rad/sec. Since ν_c is much less than Ω or ω_p the plasma may be regarded as collisionless.

Case B

If the Mariner II preliminary data are ignored, it is reasonable to assume that the field of about 40 γ measured by Cahill is the value immediately after compression of the plasma in the shock front. The lower values observed by Heppner et al (1961) are explained again by the expansion of the plasma as it moves around toward the night side of the earth. It is also reasonable to assume that the plasma velocity of about 300 km/sec measured at large distances about 45° from the anti-solar direction (Bridge et al 1961) matches fairly closely the speed of the solar wind outside the magnetosphere. There were even some indications that the probe penetrated out of the magnetosphere. The lower incident solar wind velocity for this case leads to lower values of $p_{\parallel}$ and $p_{\perp}$, namely $p_{\parallel} \approx 8 \times 10^{-9}$ dynes/cm^2 where again $p_{\perp} \approx 2p_{\parallel}$. The magnetic pressure is now about 6×10^{-9} dynes/cm^2. The plasma frequency is the same as in Case A and the Debye length nearly the same. The ion gyro angular frequency Ω is 4 rad/sec. and the plasma is, of course, collisionless.

For the conditions given either in Case A or Case B, the plasma will exhibit instabilities as described in the next section.

III. THEORETICAL PREDICTIONS

Only the briefest sketch of the theoretical methods can be given here. Emphasis will be placed on numerical results and on such approximate scaling laws as can be extracted from the theory. Instabilities are manifested as unstable waves of complex frequency $\omega = \omega_1 + i\,\omega_2$ where $\omega_2 > 0$, real wave number k, and hence complex phase velocity $u = \omega/k$. The dispersion relation which must be satisfied by the frequency and wave number of unstable transverse waves propagating along the magnetic field was given by Harris (1961) in his Eq. (30). Only such waves are used here, because there is no instability with respect to longitudinal waves propagating along the field lines and those moving in other directions are too hard to treat. Harris (1961) discussed other waves, but only in limiting conditions that do not apply here. The initial electron and ion distributions in phase space are respectively

$$f_{oe} = \frac{n_o}{a_z^3 (2\pi)^{3/2} \Lambda} \exp\left[-\frac{v_r^2}{2 a_z^2 \Lambda} - \frac{v_z^2}{2 a_z^2} \right] \tag{1a}$$

and

$$f_{oi} = \frac{n_o}{\mu^3 a_z^3 (2\pi)^{3/2} \Lambda} \exp\left[-\frac{v_r^2}{2 a_z^2 \mu^2 \Lambda} - \frac{v_z^2}{2 \mu^2 a_z^2} \right] \tag{1b}$$

where μ^2 is the ratio of the electron mass m to the proton mass M, $\Lambda = p_\perp / p_{\parallel}$, and $v_r^2 = v_x^2 + v_y^2$. Thus $2 a_z^2 m n_o = p_{\parallel}$. $\underset{\sim}{B}$ is in the z direction.

When the distributions (1) are substituted into the dispersion relation (Harris 1961) the following equation for k and $u = \omega/k$ results:

$$(1+\mu^2) k^2 (u^2 - c^2)/\omega_p^2 = 1 + \mu^{-2} \beta a_z^{-1} 2^{-\frac{1}{2}} Z(\varphi_1) + \tag{2}$$

$$+ \frac{1}{2} \Lambda Z'(\varphi_1) - \mu \beta a_z 2^{-\frac{1}{2}} Z(\varphi_2) + \frac{1}{2} \Lambda \mu^2 Z'(\varphi_2)$$

where $\omega_p^2 = 4\pi n_0 e^2(1+\mu^2)/m$, $\beta = eB/(Mck)$

$\varphi_1 = (u+\mu^{-2}\beta)/(a_z\sqrt{2})$,

$\varphi_2 = (u-\beta)/(a_z\mu\sqrt{2})$

and Z (φ) is the plasma dispersion function (Fried and Conte 1961)

$$Z(\varphi) = 2i\exp(-\varphi^2)\int_{-\infty}^{i\varphi}\exp(-t^2)\,dt \tag{3}$$

A combination of analytic and numerical methods was applied to Eq. (2). A series expansion of the right hand side of Eq. (2) was used about the points of marginal stability, where $\omega_2 = 0$. Thus some uncertainty was introduced on account of series truncation. Furthermore, the numerical work was performed only for a_z/c=0.01, while in Cases A and B a_z/c is 0.04 and 0.014 respectively. Thus approximate scaling laws derivable from the theory had to be applied. In view of these sources of error, the results soon to be stated should be taken as accurate only up to a factor of two. These results are expressed as certain parameters describing the fastest growing waves in the plasma, namely:

$f_1 = \omega_1/2\pi$ = real part of the frequency

ω_2 = imaginary part of the angular frequency = "rate of growth"

k = propagation constant

$\lambda = 2\pi/k$ = wavelength

$v_1 = \omega_1/k$ = real part of the phase velocity

$v_g = d\omega_1/dk$ = real part of the group velocity*

* v_g was evaluated for marginally stable waves rather than for the fastest growing ones.

There are two families of waves in an anisotropic plasma with $\Lambda > 1$. The fastest growing family will be designated as electron waves, because they involve almost no participation by the ions. It turns out that these waves grow so fast in the cases under consideration that they would grow beyond the linear theory in a few kilometers. Hence the waves themselves would be extremely difficult to observe. Buneman (1960) showed, however, that when unstable longitudinal waves pass well into the non-linear regime they convert most of the free energy into disordered wave energy in a few e-folding times. It is reasonable to assume that the transverse waves under study here will do the same thing, so that the end result of the electron wave instabilities is to thermalize the electrons. It is not expected that these instabilities result in turbulence because they affect the ions very little, and the ions, with their large mass, determine the macroscopic fluid velocity. Further discussion of the electron wave instabilities is deferred to the end of this section; for the present their effect will be taken into account by assuming that the electrons have been made isotropic.

Properties of the Ion Waves

The slower growing family of waves will be designated as ion waves. Unless the plasma pressure exceeds the magnetic pressure by an order of magnitude or more the ion waves form a family quite distinct from the electron waves. Then the electrons do not appreciably participate in or interfere with the ion waves. But if the magnetic field is sufficiently weak (as it is in Case A) the ion waves are sensitive to the electron distribution. One can think of this as a shielding effect. The machine calculations were for electrons and ions of equal anisotropy. Therefore in Case A the calculations were redone by hand with an isotropic electron distribution. The results could be off by more than a factor of two.

The properties of the fastest growing ion waves in cases A and B are respectively.

Case A

$$f_1 = 0.18 \text{ cps} \qquad \omega_2 = 0.17 \text{ rad/sec}$$
$$k = 0.016 \text{ km}^{-1} \qquad \lambda = 390 \text{ km} \tag{4}$$
$$u_1 = 70 \text{ km/sec} \qquad v_g = 160 \text{ km/sec}$$

Case B

$$f_1 = 0.2 \text{ cps} \qquad \omega_2 = 0.16 \text{ rad/sec}$$
$$k = 0.009 \text{ km}^{-1} \qquad \lambda = 700 \text{ km} \tag{5}$$
$$u_1 = 140 \text{ km/sec} \qquad v_g = 360 \text{ km/sec}$$

It would be desirable to know how the parameters in (4) and (5) scale when Λ, B, and a_z are changed. Approximate scaling laws are most readily found for Case B, where extensive machine calculations are available. The laws to be given are valid wherever the ratio of parallel plasma pressure, $p_{||}$ to magnetic pressure does not exceed four times the Case B value of 1.3 nor fall short of one tenth of this value. Thus the approximate laws break down before one reaches Case A, where $8\pi p_{||}/B^2 = 20$. Within the stated limitations, then, for Case B the wave number scales like $\omega_p(\Lambda - 1)^{1/2}$ unless Λ is considerably less than two; in the latter case k decreases somewhat more rapidly. For example, if $\Lambda = 1.25$ the k value is only half what would be expected on the basis of the $(\Lambda - 1)^{1/2}$ rule. (The electron waves, to be described presently, obey the rule more closely.) If the magnetic field is increased there is little change in k, but if it is reduced k decreases, slowly at first and then roughly in proportion to B. The phase velocity u_1 scales like $Ba_z\ (\Lambda - 1)^{1/2}/\Lambda$. The growth rates are roughly proportional to $a_z\omega_p k^2$.

The scaling laws for Case A appear to be similar except that k is insensitive to changes in B.

Observable Effects of the Ion Waves

Case A will be discussed first. In this case, the growth rate implies that the energy of ion anisotropy should be converted into disordered wave energy in a time of the order of 50 sec. In this time the plasma would move about 5000 km, or nearly an earth radius. For much of this period the ion waves themselves should be observable at about 0.18 cps. Thus one would expect a subregion to exist in the part of the disordered region nearest the shock front, of thickness somewhat less than an earth radius on the sunward side, where waves of velocity 60-70 km/sec, wavelength about 400 km, and frequency about 0.18 cps could be observed. The observable magnetic field spectrum after non-linear behavior has led to the formation of turbulent eddies is in more doubt. Presumably the plasma could continue to "ring" at about 0.18 cps. More likely, a probe would observe field fluctuations on a time scale given by the time needed for an eddy to sweep past it. Since the eddies would have a favored size of the order of λ for the fastest growing waves, they would be about 400 km in diameter. Sweeping past at 100 km/sec, they would pass the probe at about one each four seconds. If the field lines in an eddy are of simple form, say approximate circles, ellipses, or spirals, the field would reverse once as the eddy passes the probe. This amounts to a half cycle of A.C. signal, so that the corresponding frequency would be about 0.13 cps. Both the 0.18 cps and the 0.13 cps frequencies are fairly close to the peak at 0.1 cps observed with Pioneer I between 12.3 and 14.6 earth radii on the sunward side (Sonett et al 1960). The fit would be better if Λ were lower or if the smaller eddies were assumed to decay faster than the larger ones, which is reasonable. In the latter case one would expect to see higher frequencies nearer the shock front, and progressively lower frequencies nearer the earth and toward the night side. It should be remembered that the growth of these waves <u>per se</u> helps to explain the conversion of some of the plasma energy into magnetic energy.

The conclusions in Case B are the same except that the frequency due to the sweeping past of turbulent eddies is estimated as 0.07 cps, which is still in reasonable agreement with observations. To get closer agreement here a higher value of (or a greater density) would have to be presumed.

Electron Waves

Although there is little chance of observing the electron waves, their properties will now be given for Case B. For other examples the same scaling laws used for the ion waves in Case B may be used except as follows: k is independent of B for all values of B less than fifty times the given one; there is little departure from the $\omega_p(A-1)^{1/2}$ rule for scaling k ; and the laws are valid for arbitrarily small magnetic field strength B.

Electron Waves, Case B

$$f_1 = 350 \text{ cps} \qquad \omega_2 = 10^3 \text{ rad/sec}$$
$$k = 0.8 \text{ km}^{-1} \qquad \lambda = 7.5 \text{ km} \tag{6}$$
$$u_1 = 2.5 \times 10^3 \text{ km/sec} \qquad v_g = 4 \times 10^3 \text{ km/sec}$$

The large growth rate implies that the energy of anisotropy should be fully converted to random energy in 0.1 sec. During this time the plasma would move a distance of only 10 km.; it would still be in the thin shock region. The high group velocity implies that much of the energy may escape from the region as wave energy. This is uncertain because it is not clear if boundary conditions allow such escape-------reflection could take place. If it were possible to arrange for a detector sensitive at 350 cps to be present in the first 10 km of the shock front for appreciable fraction of a second, there is a chance that these waves could be observed. Other dissipative mechanisms in the shock might invalidate this prediction, however. The observability of these waves is then particularly sensitive to observation conditions and their existence is intimately tied up with collisionless shock theory, which is not the central subject of this paper.

In conclusion, the unstable ion waves discussed here show promise of explaining the noise spectrum observed by Sonett et al (1960) provided that the spectrum is attributed to the passage of irregularities with the mass motion of the plasma. The alternative supposition of Sonett et al was that they were observing the passage of waves through a nearly stationary plasma. Now that the plasma is known to be turbulent it would be inappropriate to think of simple Alfvén waves. The acoustic velocity is of the order of 10^3 or 10^4 km/sec., implying a wavelength of 10^4 or 10^5 km for waves with frequencies of about 0.1 cps. This becomes implausible when one compares the lengths with the dimensions of the region or tries to think of generation mechanisms.

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